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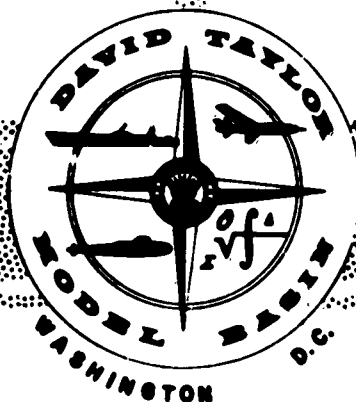
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435607

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DEPARTMENT OF THE NAVY

ON THE THEORY OF STABILITY OF A PROLATE SPHEROIDAL SHELL  
UNDER UNIFORM EXTERNAL PRESSURE

(K Teorii Ustoichivosti Vytyanutoi Ellipsoidal'noi  
Obolochki Vrashcheniya Pri Vneshnem Ravnomernom Davlenii)

by

R. G. Surkin

Kazanskii Filial Akademii Nauk SSSR,  
Seriya Fiziko-Matematicheskikh i  
Tekhnicheskikh Nauk, No. 7, 1955

STRUCTURAL MECHANICS LABORATORY

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APPLIED  
MATHEMATICS

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VIBRATION

January 1964

Translation 317

NO. OTS

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ON THE THEORY OF STABILITY OF A PROLATE SPHEROIDAL SHELL  
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Translated by Barry I. Hyman

January 1964

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## PREFACE

This paper is the only known published work on the nonlinear theory of stability of prolate spheroids under external pressure. An extensive search by the translator revealed that the particular issue of the journal (Kazanskii Filial Akademii Nauk SSSR, Seriya Fiziko-Matematicheskikh i Tekhnicheskikh Nauk, No. 7, 1955) in which this paper appeared was not available in this country. A copy of the article as it appeared in the journal was obtained after direct correspondence with the author; and this translation serves to make this work available on a wide scale.

The translator wishes to acknowledge his indebtedness to Mrs. P. Hale of Virginia Polytechnic Institute and to Dr. B. Nakonechny of the David Taylor Model Basin for their valuable assistance in the translation of this paper.

## NOTATION

D	Bending rigidity, $\frac{Et^3}{12(1-\nu^2)}$
E	Young's modulus
K	Tensile rigidity, $\frac{Et}{1-\nu^2}$
$p_e$	Critical pressure of the shell according to the linear theory
$p_k$	Pressure at which the stable and unstable states of equilibrium coincide, i.e., at which the first and second variations of the energy functional $\Phi$ are equal to zero
$p_m$	Critical pressure of the shell according to the nonlinear theory, i.e., the lower limit of all values of the pressure $p$ for which the energy of the "nonlinear" state is smaller than the energy of the "zero" state
$T_1, T_2$	Additional stresses in the middle surface (after snapping)
$T_{01}, T_{02}$	Stresses in the middle surface of the shell for the membrane state (prior to snapping)
t	Thickness of shell

$u, v$  Projections of the displacement of a point of the middle surface along the lines  $\alpha$  and  $\beta$

$W$  Specific work of the external load

$w$  Projection of the displacement on the inward normal to the middle surface

$u_x = \frac{\partial u}{\partial x}, v_y = \frac{\partial v}{\partial y}, w_x = \frac{\partial w}{\partial x}, \dots$  Corresponding partial derivatives of the displacements

$\kappa_1, \kappa_2, \kappa_{12}$  Curvature parameters

$\alpha$  and  $\beta$  Gaussian coordinates of the middle surface of a shell of revolution along the meridians and parallels;  $R_1$  and  $R_2$  are their radii of curvature

$\delta$   $\frac{R_1}{R_2}$

$\epsilon_1, \epsilon_2$  Relative elongations in the directions of the coordinate lines  $\alpha$  and  $\beta$

$\nu$  Poisson's ratio

$\tau$   $\frac{1}{\sqrt{12}} \frac{t}{R_1}$

$\omega$  Angle of displacement between the coordinate lines  $\alpha$  and  $\beta$ .

## ABSTRACT

The Rayleigh-Ritz method is used to solve the problem of stability of prolate spheroidal shells under uniform external pressure. Nonlinear terms are retained in the analysis. The "equal energy" load and the minimum post-buckling load are determined for several cases that demonstrate the effect of varying the eccentricity of the generating ellipse.

## INTRODUCTION

This paper deals with the possibility of the local loss of stability of a prolate spheroid, under the influence of uniform external normal pressure distributed over the entire shell. Large displacement theory, which allows for snap-through buckling, is used.

The present work represents a generalization of the well-known theory of snapping of shells<sup>1</sup> for the case of prolate spheroids.

Here the critical pressures  $p_m$  and  $p_k$  are determined by the energy method, as was done for the spherical shell. This means that the pressure  $p_m$  corresponds to equal levels of total energy of the shell in the "zero" and "nonlinear" states; the pressure  $p_k$  corresponds to the case where the stable and unstable states of equilibrium coincide, at which point the energy function  $\Phi$  has a parabolic point, i.e., the first and second variations of  $\Phi$  are equal to zero.

The problem under discussion is solved in a general form, and in addition, certain numerical examples are investigated.

### 1. DETERMINATION OF $p_m$

The solution of the problem is carried out for the assumption that the center of the snap lies on the equator of the shell and in a plan view the region of the snap resembles the form of an ellipse. This assumption is reasonable, since in the investigation of the local loss of stability of a geometrically perfect spheroid shell the weakest part is in the region of the equator. In the regions remote from the equator the curvature of the shell is greater; consequently, the stiffness of the shell will also be greater.

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<sup>1</sup>References are listed on page 16.



If, in addition to the fundamental "zero" state of equilibrium of the shell, it is possible that for the same loading there is a stable position of equilibrium after snapping, then the total energy of the shell must also be at a minimum in this final state. Thus, the problem is reduced to a minimization of the functional:

$$\Phi = \int_{(w)} \left\{ \frac{K}{2} [(e_1^0 + e_1)^2 + (e_2^0 + e_2)^2 + 2\nu(e_1^0 + e_1)(e_2^0 + e_2) + \frac{(1-\nu)}{2} w^2] + \right. [1.1]$$

$$\left. + \frac{D}{2} [x_1^2 + x_2^2 + 2\nu x_1 x_2 + 2(1-\nu)x_{12}] - W \right\} dx dy,$$

where

$$R_1 da = dx, \quad R_2 d\beta = dy;$$

and

$$e_1^0 = -\frac{w_0}{R_1}, \quad e_2^0 = -\frac{w_0}{R_2}$$

are the strains in the middle surface of the shell before snapping for the assumption that prior to the local loss of stability the shell is in a membrane state;  $w_0$  is the original deflection in the membrane state; [1.1] is integrated over the entire middle surface of the shell; and the normal to the shell is considered to be directed inward.

The relative displacements and curvature parameters can be written in the form:

$$t_1 = u_x + \frac{1}{2} w_x^2 - w/R_1, \quad t_2 = v_y + \frac{1}{2} w_y^2 - w/R_2;$$

$$w = v_x + u_y + w_x w_y; \quad x_1 = w_{xx}, \quad x_2 = w_{yy}, \quad x_{12} = w_{xy}. \quad [1.2]$$

Taking into account the condition for equilibrium of an element of the shell in the direction normal to the middle surface in the presence of external uniform pressure on the shell

$$\frac{T_{\theta 1}}{R_1} + \frac{T_{\theta 2}}{R_2} = -p, \quad [1.3]$$

where

$$T_{\theta 1} = K(e_1^0 + w_{e1}^0), \quad T_{\theta 2} = K(e_2^0 + w_{e2}^0), \quad [1.4]$$

we present the work per unit area of the external forces in the form:

$$W = p(w + w_0) = -\left(\frac{T_{\theta 1}}{R_1} + \frac{T_{\theta 2}}{R_2}\right)(w + w_0) =$$

$$= -K \left[ e_1^0 \left( -e_1^0 + \frac{w}{R_1} + \nu \frac{w}{R_2} \right) + e_2^0 \left( -e_2^0 + \nu \frac{w}{R_1} + \frac{w}{R_2} \right) - 2\nu e_1^0 e_2^0 \right], \quad [1.5]$$

here we include in the energy functional only the work of the normal pressure on the shell since, by virtue of the boundary conditions for all the cases which are considered further, the work of the reaction of the remaining part of the shell on the boundary of the snap region is equal to zero.

Considering expressions [1.2] - [1.5], we can reduce the functional [1.1] to the form:

$$\begin{aligned} \Phi = & \int \int_{(\omega)} \left\{ \frac{1}{2K(1-\nu)} [-(T_{01}^2 + T_{02}^2) + 2\nu T_{01} T_{02}] + u_x T_{01} + v_y T_{02} \right\} dx dy + \\ & + \int \int_{(\omega)} \left\{ \frac{K}{2} [s_1^2 + s_2^2 + 2\nu s_1 s_2 + w_x^2 \frac{T_{01}}{K} + w_y^2 \frac{T_{02}}{K} + \frac{(1-\nu)}{2} s^2] + \right. \\ & \left. + \frac{D}{2} [x_1^2 + x_2^2 + 2\nu x_1 x_2 + 2(1-\nu) x_{12}^2] \right\} dx dy. \end{aligned} \quad [1.6] \quad *$$

At the same time, it is assumed that one can neglect the change in  $R_1$  and  $R_2$  in the region of the snap, since the size of the snap region is small in comparison to the size of the shell.

Obviously, the total energy in the first form of equilibrium (before snapping) is equal to

$$\Phi_0 = \int \int_{(\omega)} \frac{1}{2K(1-\nu)} [-(T_{01}^2 + T_{02}^2) + 2\nu T_{01} T_{02}] dx dy,$$

where the integral is taken over the entire shell. Then, the problem is reduced to the minimization of the functional

$$\Phi' = \Phi - \Phi_0.$$

On the assumption that, in the snap region,  $T_{01}$  and  $T_{02}$  are constant, the expression

$$\varphi = \int \int_{(\omega)} (u_x T_{01} + v_y T_{02}) dx dy,$$

which enters into [1.6], is equal to zero in virtue of the boundary conditions, since we put

$$u=0, v=0, w_x=0, w_y=0 \text{ for } \alpha=\alpha_0 \text{ and } \beta=\beta_0.$$

Thus, we have

$$\begin{aligned} \Phi' = & \frac{K}{2} \int \int_{(\omega)} \left\{ s_1^2 + s_2^2 + 2\nu s_1 s_2 + \frac{(1-\nu)}{2} s^2 + \nu^2 R_1^2 [x_1^2 + x_2^2 + \right. \\ & \left. + 2\nu x_1 x_2 + 2(1-\nu) x_{12}^2] + w_x^2 \frac{T_{01}}{K} + w_y^2 \frac{T_{02}}{K} \right\} dx dy. \end{aligned} \quad [1.7]$$

Here the integration is carried out only over the region of buckling  $S'$ , since the quantities characterizing the snap may be different from zero

\*The terms  $w_x^2$  and  $w_y^2$  were incorrectly printed as  $e_{13}^2$  and  $e_{23}^2$  in the original.

only in the region

$$0 \leq \alpha \leq \alpha_0 \text{ and } 0 \leq \beta \leq \beta_0.$$

We introduce the new variables

$$\frac{x^2}{x_0^2} = \xi, \frac{y^2}{y_0^2} = \eta, \quad 0 \leq \xi \leq 1, \quad 0 \leq \eta \leq 1, \quad [1.8]$$

and the notation

$$x_0 = R_1 \alpha_0, \quad y_0 = R_2 \beta_0, \quad \xi = \frac{R_1}{R_2}, \quad [1.9]$$

where  $x_0$  and  $y_0$  are the linear dimensions of the snap region in the directions of the meridian and equator of the shell. In the following, we will assume that the contour of the snap region is determined by the ellipse

$$\xi + \eta = 1.$$

We choose the displacements in the general form:

$$u = \rho_1 \lambda \alpha_0^2 R_1 h(\xi, \eta), \quad v = \rho_2 \lambda \alpha_0^2 \beta_0 R_1 j(\xi, \eta), \quad [1.10]$$

$$w = \lambda \alpha_0^2 R_1 g(\xi, \eta),$$

where

$$h(\xi, \eta), j(\xi, \eta) \text{ and } g(\xi, \eta)$$

are some functions of  $\xi, \eta$ , characterizing the displacements and which should satisfy the boundary conditions, i.e.,

$$h(\xi, \eta) = j(\xi, \eta) = 0 \text{ for } \xi = \eta = 0 \text{ and } \xi + \eta = 1, \quad [1.11]$$

$$g(\xi, \eta) = 1 \text{ for } \xi = \eta = 0 \text{ and } g(\xi, \eta) = 0 \text{ for } \xi + \eta = 1,$$

where  $\rho_1, \rho_2$ , and  $\lambda$  are unknown parameters. The magnitude of the angles  $\alpha_0$  and  $\beta_0$ , which determine the extent of the buckled region, are also unknown. However, a simple relation exists between these angles.

In fact, according to our assumption, the contour of the buckled region projected onto a plane tangent to the spheroid at the equator is the ellipse (see Figure 1)

$$\frac{x^2}{x_0^2} + \frac{y^2}{y_0^2} = 1. \quad [1.12]$$

The equation of the ellipsoid formed by rotating an ellipse with semiaxes  $a$  and  $b$  about the axis  $O_1 x_1$  has the form

$$\frac{x_1^2}{b^2} + \frac{y_1^2 + z_1^2}{a^2} = 1. \quad [1.13]$$

If the contour of the snap region lies in the  $\pi$ -plane parallel to the plane  $x_1Oy_1$ , and is separated from it by a distance  $z_1 = a-d$ , then

$$z_1^2 = a^2 - 2ad + d^2, \quad [1.14]$$

where the distance  $d \ll a$ .

Substituting [1.14] into [1.13], we obtain the equation of an ellipse lying in the  $\pi$ -plane:

$$\frac{x_1^2}{2 \frac{d\delta^2}{a} \left(1 - \frac{d}{2a}\right)} + \frac{y_1^2}{2ad \left(1 - \frac{d}{2a}\right)} = 1.$$

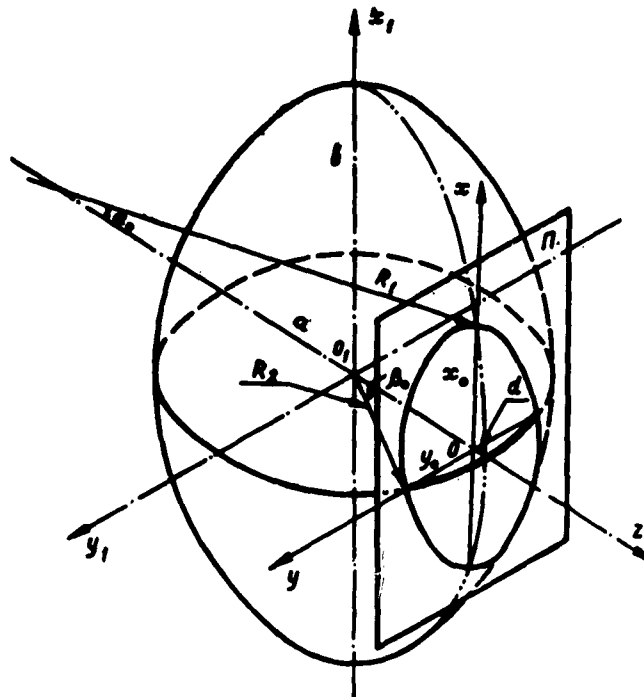


Figure 1

Comparing this equation with [1.12], we find:

$$x_0^2 : y_0^2 = b^2 : a^2 = R_1 : R_2 = \delta.$$

On the other hand

$x_0 = R_1 \alpha_0$ ,  $y_0 = R_2 \beta_0$ , where  $\alpha_0$  and  $\beta_0$  are small angles. Therefore

$$R_1^2 \alpha_0^2 : R_2^2 \beta_0^2 = \delta, \alpha_0^2 : \beta_0^2 = 1 : \delta. \quad [1.15]$$

Thus, in place of the quantities  $\alpha_0$  and  $\beta_0$ , we may introduce a single unknown parameter, namely

$$r = \alpha_0 \beta_0. \quad [1.16]$$

We take into account that

$$T_{02} = T_{01} \left(2 - \frac{1}{\delta}\right), \quad [1.17]$$

and introduce the new symbols

$$\begin{aligned} e_0 &= -T_{02} : K = |\alpha_{02}| (1 - v^2) : E, \quad [1.18] \\ A_1 &= \frac{1}{v^2} \left[ A_{11} + \frac{(1-v)}{2} \delta A_{12} \right], \quad A_2 = A_{21} + \frac{1-v}{2\delta} A_{22}, \\ A_3 &= -\frac{1+v}{v^2} A_{31}, \quad A_4 = \frac{1+2v+\delta^2}{4\delta^2} \int_0^1 \int_0^1 \frac{1}{V \xi \eta} g^2 d\xi d\eta, \\ A_5 &= \frac{1}{\delta} \int_0^1 \int_0^1 [2vh_\eta J_\eta + (1-v) h_\eta J_\xi] d\xi d\eta, \quad A_6 = -\frac{\delta+v}{\delta} A_{22}, \\ B_1 &= \frac{2}{v^2} [B_{11} + v\delta B_{12} + (1-v)\delta B_{13}], \quad B_2 = 2 \left( B_{21} + \frac{v}{\delta} B_{22} + \frac{1-v}{\delta} B_{23} \right), \\ B_3 &= -\frac{1}{v^2} \int_0^1 \int_0^1 \left[ (1+v\delta) \sqrt{\frac{\xi}{\eta}} g g_\xi^2 + (\delta+v)\delta \sqrt{\frac{\eta}{\xi}} g g_\eta^2 \right] d\xi d\eta, \\ D_1 &= \int_0^1 \int_0^1 \left[ \frac{1}{v^2} \sqrt{\frac{\xi}{\eta}} g_\xi^2 + \frac{\eta}{V \xi} g_\eta^2 + \frac{2}{\delta} V \xi \eta g_\xi^2 g_\eta^2 \right] d\xi d\eta, \\ C &= \int_0^1 \int_0^1 \left[ \frac{1}{\delta} \frac{1}{V \xi \eta} (2\xi g_\eta + g_\xi)^2 + \frac{\delta}{V \xi \eta} (2\eta g_\eta + g_\eta)^2 + \right. \\ &\quad \left. + \frac{2v}{V \xi \eta} (2\xi g_\eta + g_\xi)(2\eta g_\eta + g_\eta) + 8(1-v)V \xi \eta g_\xi^2 \right] d\xi d\eta, \\ A &= \frac{1}{V \delta} \int_0^1 \int_0^1 \left[ \frac{1}{2\delta-1} \sqrt{\frac{\xi}{\eta}} g_\xi^2 + \sqrt{\frac{\eta}{\xi}} g_\eta^2 \right] d\xi d\eta. \end{aligned}$$

$$\begin{aligned}
A_{11} &= \int_0^1 \int_0^1 V \frac{\xi}{\eta} h_{\xi}^2 d\xi d\eta, & A_{12} &= \int_0^1 \int_0^1 V \frac{\eta}{\xi} h_{\eta}^2 d\xi d\eta, \\
A_{31} &= \int_0^1 \int_0^1 \frac{1}{V \eta} g h_{\xi} d\xi d\eta, & B_{11} &= \int_0^1 \int_0^1 \frac{\xi}{V \eta} h_{\xi} g_{\xi}^2 d\xi d\eta, \\
B_{12} &= \int_0^1 \int_0^1 V \eta h_{\xi} g_{\eta}^2 d\xi d\eta, & B_{13} &= \int_0^1 \int_0^1 V \eta h_{\eta} g_{\eta} d\xi d\eta,
\end{aligned} \tag{1.19}$$

where  $A_{21}$ ,  $A_{22}$ ,  $A_{32}$ ,  $B_{21}$ ,  $B_{22}$ ,  $B_{23}$  are obtained respectively from  $A_{11}$ ,  $A_{12}$ ,  $A_{31}$ ,  $B_{11}$ ,  $B_{12}$ ,  $B_{13}$  by replacing  $h$  by  $j$  and  $\xi$  by  $\eta$ .

Using the symbols just introduced, after lengthy but in reality simple calculations, we can represent the functional [1.7] in the following form:

$$\begin{aligned}
\Phi^* &= \frac{2\Phi'}{KR_1^2} = r^2 \lambda^2 [\rho_1^2 A_1 + \rho_2^2 A_2 + \rho_1 A_3 + A_4 + \rho_1 \rho_2 A_5 + \rho_2 A_6 + \\
&+ \lambda \rho_1 B_1 + \lambda \rho_2 B_2 + \lambda B_3 + \lambda^2 D_1] + \tau^2 \lambda^2 r C - e_0 \lambda^2 r^2 A.
\end{aligned} \tag{1.20}$$

Here  $\lambda$ ,  $\rho_1$ ,  $\rho_2$ , and  $r$  are unknown parameters characterizing the snap region and  $e_0$ . To minimize the functional [1.20] by the Ritz-Timoshenko method for the determination of  $p_m$ , which is the lowest limit of all values of  $p$  for which the energy of the "nonlinear" state is less than the energy of the "zero" state, it is necessary to fulfill the following conditions:

$$\Phi^* = 0, \Phi_{\lambda}^* = 0, \Phi_{\rho_1}^* = 0, \Phi_{\rho_2}^* = 0, \Phi_r^* = 0. \tag{1.21}$$

Hence, we obtain equations for the determination of  $\rho_1$ ,  $\rho_2$ ,  $\lambda$ ,  $r$ , and

$$\begin{aligned}
e_{om}: & \left. \begin{aligned} 2\rho_1 A_1 + \rho_2 A_5 + \lambda B_1 + A_3 &= 0, \\ \rho_1 A_5 + 2\rho_2 A_2 + \lambda B_2 + A_6 &= 0, \\ \rho_1 B_1 + \rho_2 B_2 + 2\lambda D_1 + B_3 &= 0, \end{aligned} \right\} \tag{1.22}
\end{aligned}$$

$$r^2 [A_4 - \rho_1^2 A_1 + \rho_2^2 A_2 - \rho_1 \lambda B_1 + \rho_2 A_6 - \lambda^2 D_1] = C \tau^2 \tag{1.23}$$

$$e_{om} = \frac{2C}{rA} \tau^2, \tag{1.24}$$

where

$$\tau = \frac{1}{V^{12}} \frac{t}{R_1} = \frac{1}{V^{12}} \frac{t}{R_2}.$$

As is evident for the numerical solution of a particular problem, it is necessary to determine the values of the functionals  $A_1$ ,  $A_2$ , ...,  $B_1$ , ..., and  $A$ . The latter (according to [1.19]) depend only on the form of the functions for the displacements  $h$ ,  $j$ , and  $g$ . Proper selection of the

displacement functions satisfying the boundary conditions [1.11] obviously guarantees a more dependable solution to the problem.

From our investigation of six alternate forms for the displacements, we retained the one that, in the final analysis, gave the minimum value for the pressure  $p_m$  at the values  $\delta = 1, 2, 3, 4$ :

$$\begin{aligned} h(\xi, \eta) &= e^{-n(\xi+\eta)} [1 - k(\xi + \eta) - k_2(\xi^2 + \eta^2)], \\ j(\xi, \eta) &= e^{-n(\xi+\eta)} [1 - k_1(\xi + \eta) - k_4(\xi^2 + \eta^2)], \\ g(\xi, \eta) &= e^{-n(\xi+\eta)} [1 - k_3(\xi + \eta)]. \end{aligned} \quad [1.25]$$

Here  $k, k_1, k_2, k_3, k_4$ , and  $n$  are quantities to be determined, where we will assume that  $n$  will be chosen a number such that on the boundary of the buckled region ( $\xi + \eta = 1$ ), the deflection becomes negligible.

Further, using formulas [1.19], we compute the coefficients  $A_1, A_2, \dots, B_1, \dots$ , and  $A$  of the energy functional [1.20]. In addition, in formulas [1.19], the limits of integration are taken from 0 to  $\infty$  since, for the assumed form of the displacements [1.25], the displacements and stresses are negligible on the boundary of the snap region ( $\xi + \eta = 1$ ).

Omitting the detailed calculations, we can write the coefficients of the energy functional in their final form:

$$\begin{aligned} A_{11} &= \frac{6}{64} \cdot \frac{\pi}{n} \left( 1 - \frac{k}{n} + \frac{k^2}{n^2} + \frac{1}{4} \frac{k_2}{n^2} + \frac{17}{8} \frac{kk_2}{n^3} + \frac{185}{64} \frac{k_2^2}{n^4} \right), \\ A_{12} &= \frac{1}{32} \cdot \frac{\pi}{n} \left( 1 - \frac{k}{n} + \frac{k^2}{n^2} - \frac{3}{4} \frac{k_2}{n^2} + \frac{21}{8} \frac{kk_2}{n^3} + \frac{225}{64} \frac{k_2^2}{n^4} \right), \\ A_{31} &= -\frac{\pi}{16n} \left( 2 - 2 \frac{k}{n} + \frac{kk_2}{n^2} - \frac{9}{4} \frac{k_2}{n^2} + \frac{9}{4} \frac{k_2^2}{n^3} \right), \\ A_4 &= \frac{(1 + 2\delta + \delta^2)}{16\delta^3} \cdot \frac{\pi}{n} \left( 2 - 2 \frac{k_2}{n} + \frac{k_2^2}{n^2} \right), \\ A_5 &= \frac{\pi(1+\delta)}{16\delta n} \left( 2 - \frac{k}{n} - \frac{k_1}{n} + 2 \frac{kk_1}{n^2} - \frac{3}{4} \frac{k_2}{n^2} - \frac{3}{4} \frac{k_4}{n^2} + \frac{21}{8} \frac{kk_2}{n^3} + \frac{21}{8} \frac{kk_4}{n^3} \right), \\ B_{11} &= \frac{\pi}{54} \left( \frac{k_2}{n} - \frac{k}{n} - \frac{kk_2}{n^2} - \frac{5}{3} \frac{k_2}{n^2} + \frac{1}{18} \frac{k_2^2}{n^3} - \frac{2}{9} \frac{k_2^3}{n^4} \right), \\ B_{12} &= \frac{\pi}{9 \cdot 54} \left[ 9 - 9 \frac{k}{n} + 9 \frac{k_2}{n} - 3 \frac{kk_2}{n^2} + 3 \frac{k_2^2}{n^3} - 2 \frac{kk_2^2}{n^4} - \right. \\ &\quad \left. - \frac{5}{2} \left( 3 \frac{k_2}{n^2} - \frac{k_2^2}{n^3} + \frac{k_2^3}{n^4} \right) \right], \\ B_{13} &= -\frac{\pi}{18 \cdot 54} \left( 9 + 6 \frac{kk_2}{n^2} + 3 \frac{k_2^2}{n^3} - 2 \frac{kk_2^2}{n^4} + \frac{3}{2} \frac{k_2}{n^2} + 4 \frac{k_2^2}{n^3} - \frac{5}{2} \frac{k_2^3}{n^4} \right), \\ B_3 &= \frac{\pi}{162} \cdot \frac{(1 + 2\delta + \delta^2)}{\delta^3} \left( 2 \frac{k_2^2}{n^2} - 3 \frac{k_2^3}{n^3} - 9 \right), \end{aligned}$$

$$\begin{aligned}
D_1 &= \frac{\pi n}{8192n^3} (38^3 + 28 + 3) \left( 32 + 32 \frac{k_1}{n} + 48 \frac{k_2^2}{n^2} + 8 \frac{k_3^3}{n^3} + 5 \frac{k_4^4}{n^4} \right), \\
C &= \frac{\pi n}{88} (38^3 + 28 + 3) \left( 1 + \frac{k_1}{n} + \frac{k_2^2}{n^2} \right), \\
A &= \frac{\pi}{8} \frac{\sqrt{8}}{28-1} \left( 2 + \frac{k_2^2}{n^2} \right). \quad [1.26]
\end{aligned}$$

The coefficients  $A_{21}$ ,  $A_{22}$ ,  $A_{32}$ ,  $B_{21}$ ,  $B_{22}$ , and  $B_{23}$  are obtained from  $A_{11}$ ,  $A_{12}$ ,  $A_{31}$ ,  $B_{11}$ ,  $B_{12}$ , and  $B_{13}$  by replacing  $k$  and  $k_3$  respectively by  $k_1$  and  $k_4$ .

These coefficients are functions of  $\delta$  and of the unknown parameters  $\frac{k}{n}$ ,  $\frac{k_1}{n}$ ,  $\frac{k_2}{n}$ ,  $\frac{k_3}{n^2}$ , and  $\frac{k_4}{n^2}$ . The latter in our case are determined by means of successive selection and, for  $\delta = 1$ , proved to be equal to

$$\frac{k}{n} = \frac{k_1}{n} = 0.150; \quad \frac{k_2}{n} = -0.545; \quad \frac{k_3}{n^2} = \frac{k_4}{n^2} = -0.055. \quad [1.27]$$

To simplify the computations we also used [1.27] for  $\delta \neq 1$ . The numerical determination of  $p_m$  was carried out in the following manner:

(a) For a given  $\delta$  ( $\delta = 1, 2, 3, 4$ ) and the values of the parameters

$\frac{k}{n}$ ,  $\frac{k_1}{n}$ , ... from [1.27], the coefficients of the energy functional  $A_1, A_2, \dots, B_1, \dots, A$  are calculated according to [1.19] and [1.26].

(b) The values  $\rho_1$ ,  $\rho_2$ , and  $\lambda$  are determined from Equations [1.22].

In this case,  $\rho_1$  and  $\rho_2$  are not dependent on the order of the decay  $n$  in the displacement functions; however,  $\lambda$  does depend on  $n$ .

(c) The obtained values of  $\rho_1$ ,  $\rho_2$ , and  $\lambda$  are substituted in Equation [1.23], and we calculate the parameter  $r = \alpha_0 \beta_0$ , which depends on  $n$  and  $\frac{t}{R_2}$ .

Knowing  $r$  and taking into account [1.15], we determine without difficulty the values of the small solid angles of the buckle

$$\alpha = \sqrt{\frac{r}{Vt}}, \quad \beta = \sqrt{rVt}. \quad [1.28]$$

(d) For a known  $r$ , we compute  $e_{om}$  according to formula [1.24].

Then  $\sigma_{01}^m$ ,  $\sigma_{02}^m$ , and  $p_m$  are determined. Considering [1.3], [1.17], and [1.18] and assuming  $r = r^* \tau$  (where  $r^*$  is a numerical coefficient), we write these in the general form



$$\alpha_{01}^m = -\frac{1}{\sqrt{3}(2\delta-1)} \frac{C}{r^2 A} \frac{E}{1-\nu^2} \cdot \frac{t}{R_2}, \quad [1.29]$$

$$\alpha_{02}^m = -\frac{1}{\delta\sqrt{3}} \frac{C}{r^2 A} \frac{E}{1-\nu^2} \cdot \frac{t}{R_2},$$

$$p_m = \frac{2}{\sqrt{3}(2\delta-1)} \frac{C}{r^2 A} \cdot \frac{E}{1-\nu^2} \cdot \frac{t^3}{R_2^3}. \quad [1.30]$$

(e) The maximum deflection in the center of the buckled region is determined from [1.10] and [1.25] for  $\xi = \eta = 0$ :

$$w_{\max} = \lambda \alpha_0^2 R_1.$$

Since

$$\alpha_0^2 = \frac{r}{\sqrt{t}} \text{ and } r = r^* n \frac{t}{\sqrt{12} R_1}, \quad \lambda = \lambda^* \frac{1}{n},$$

(where  $r^*$  and  $\lambda^*$  are numerical coefficients), we obtain

$$\frac{w_{\max}}{t} = \frac{\lambda^* r^*}{\sqrt{12\delta}}. \quad [1.31]$$

Thus the maximum relative displacement in the center of the buckled region does not depend on the relative thickness of the shell  $t/R_2$  but on the order of the decay  $n$ .

In Table 1 the values of the critical pressure  $p_m$  and the dimensions of the buckled region for different values of  $\delta$  are given for

$$\frac{k}{n} = \frac{k_1}{n} = 0.150, \frac{k_2}{n} = -0.545, \frac{k_3}{n^2} = \frac{k_4}{n^2} = -0.055 \text{ and } \nu = 0.3.$$

Table 1 shows that the solid angles of the buckled region  $\alpha_0$  and  $\beta_0$  depend on  $n$  and  $\frac{t}{R_2}$ . Supposing that  $\frac{t}{R_2} = \frac{1}{900}$  and assuming that for  $n=5$  (or also for  $n=4$ ) the buckle is very small on the boundary of the snap region, we calculate the values of the small angles  $\alpha_0$  and  $\beta_0$ ; see Table 2.

## 2. DETERMINATION OF $p_k$

For the pressure equal to  $p_k$  we have a parabolic point for  $\Phi^*$  on the energy-deflection graph, i.e., the first and second variations of  $\Phi^*$  [1.20] are equal to zero.

TABLE 1

$\delta = \frac{R_1}{R_2}$	1	2	3	4
$\rho_1$	1.2225	1.2243	1.2257	1.2267
$\rho_2$	1.2225	1.2216	1.2207	1.2208
$\lambda n$	4.2619	4.2581	4.2563	4.2558
$r \cdot \frac{1}{n\tau}$	5.9933	7.7872	10.086	11.640
$\frac{x_o}{y_o}$	1	$\sqrt{2}$	$\sqrt{3}$	2
$\frac{\alpha_o}{\beta_o}$	1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{1}{2}$
$\alpha_o \sqrt{\frac{R_2}{nt}}$	1.3153	1.2971	1.2960	1.2610
$\beta_o \sqrt{\frac{R_2}{nt}}$	1.3153	1.7830	2.2457	2.5220
$e_{om} \frac{R_2}{t}$	0.2205	0.2135	0.1878	0.1816
$ \sigma_{01}^m  \frac{R_2}{Et}$	0.2423	0.1564	0.1238	0.1140
$ \sigma_{02}^m  \frac{R_2}{Et}$	0.2423	0.2346	0.2064	0.1996
$\frac{p_m R_2^2}{Et^2}$	0.4446	0.3128	0.2476	0.2280
$\frac{w_{max}}{t}$	7.37	7.25	7.15	7.14

TABLE 2

$\delta$		1	2	3	4
$\alpha_o$	n = 4	5°02'	4°58'	4°57'	4°48'
	n = 5	5°40'	6°51'	8°35'	9°55'
$\beta_o$	n = 4	5°02'	5°33'	5°30'	5°23'
	n = 5	5°40'	7°55'	9°36'	11°04'

Therefore,  $p_k$  is determined from the equations

$$\begin{aligned} \Phi_\lambda^* = 0, \quad \Phi_{\rho_1}^* = 0, \quad \Phi_{\rho_2}^* = 0, \quad \Phi_r^* = 0, \\ \begin{vmatrix} \Phi_{\lambda\lambda}^* & \Phi_{\lambda\rho_1}^* & \Phi_{\lambda\rho_2}^* & \Phi_{\lambda r}^* \\ \Phi_{\rho_1\lambda}^* & \Phi_{\rho_1\rho_1}^* & \Phi_{\rho_1\rho_2}^* & \Phi_{\rho_1 r}^* \\ \Phi_{\rho_2\lambda}^* & \Phi_{\rho_2\rho_1}^* & \Phi_{\rho_2\rho_2}^* & \Phi_{\rho_2 r}^* \\ \Phi_{r\lambda}^* & \Phi_{r\rho_1}^* & \Phi_{r\rho_2}^* & \Phi_{rr}^* \end{vmatrix} = 0 \end{aligned} \quad [2.1]$$

where  $\Phi_\lambda^*, \Phi_{\rho_1}^*, \dots, \Phi_r^*, \Phi_{\lambda\lambda}^*, \dots, \Phi_{rr}^*$  are the corresponding partial derivatives with respect to the parameters  $\lambda, \rho_1, \rho_2$ , and  $r$ . After certain transformations in the first four equations [2.1], we obtain:

$$\begin{aligned} 2\rho_1 A_1 + \rho_2 A_2 + \lambda B_1 + A_3 &= 0 \\ \rho_1 A_2 + 2\rho_2 A_3 + \lambda B_2 + A_4 &= 0 \\ \rho_1 B_1 + \rho_2 B_2 + 2\lambda D_1 + B_3 + \frac{2}{3} \frac{(2\tau^2 C - e_{ok} r A)}{r^2 \lambda} &= 0. \end{aligned} \quad [2.2]$$

$$r^2 [\rho_1^2 A_1 + \rho_2^2 A_2 + \rho_1 A_3 + A_4 + \rho_1 \rho_2 A_5 + \rho_2 A_6 - \lambda^2 D_1] = \tau^2 C. \quad [2.3]$$

The calculation of the fourth order determinant in [2.1] does not present particular difficulty since  $\Phi_{\rho_1 r}^*$  and  $\Phi_{\rho_2 r}^*$  are equal to zero. Calculation of the fourth order determinant in [2.1] gives us:

$$\begin{aligned} [(2\tau^2 C - e_{ok} r A)^2 + (2\tau^2 C - e_{ok} r A)(\tau^2 C - e_{ok} r A)](A_5^2 - 4A_1 A_2) - \\ - r^2 \lambda^2 (\tau^2 C - e_{ok} r A) [B_1 B_2 A_3 - A_1 B_2^2 - A_2 B_1^2 - D_1 (A_5^2 - 4A_1 A_2)] = 0. \end{aligned} \quad [2.4]$$

Equations [2.2], [2.3], and [2.4] are completely sufficient for determining the five unknowns  $\lambda, \rho_1, \rho_2, r$ , and  $e_{ok}$ . For the solution of the problem we will make certain transformations.

In the third equation of [2.2] we introduce the notation

$$\frac{2}{3} \frac{(2\tau^2 C - e_{ok} r A)}{r^2 \lambda} = s. \quad [2.5]$$

Then  $\rho_1$ ,  $\rho_2$ , and  $\lambda$  are determined from Equations [2.2] and are linear functions of  $\epsilon$ . Further, for known  $\rho_1$ ,  $\rho_2$ , and  $\lambda$ , from Equation [2.3] we determine  $\frac{\tau^2 C}{r^2}$ , which will be a quadratic function of  $\epsilon$ . Using the notation

[2.5], we transform Equation [2.4]:

$$\lambda \epsilon^2 - \frac{1}{3} \frac{\tau^2 C}{r^2} \epsilon + \left[ \frac{2}{9} \frac{\tau^2 C}{r^2} \lambda - \frac{1}{3} \lambda^2 \right] \frac{(B_1 B_2 A_3 - A_1 B_2^2 - A_2 B_1 - D_1 (A_3^2 - 4 A_1 A_2))}{A_3^2 - 4 A_1 A_2} = 0. \quad [2.6]$$

After we substitute the values  $\lambda$  and  $\frac{\tau^2 C}{r^2}$  obtained for the particular values

of  $\delta = 1, 2, 3, 4$ , Equation [2.6] becomes a cubic equation involving  $\epsilon$ . A cubic equation is solvable by well-known methods, and all three of its roots can be determined. Computations showed that for a given  $\delta$ , only the smallest root of Equation [2.6] was applicable to the determination of  $p_k$ .

For known  $\epsilon$ , we easily calculate  $e_{ok}$  from Equation [2.5]. Knowing  $e_{ok}$ , we obtain  $p_k$  from formulas [1.18], [1.17], and [1.3]. We have calculated the value of  $p_k$  for spheroids with different elongations, i.e., for the particular cases  $\delta = 1, 2, 3, 4$ . The results of the computations are given in Table 3.

Here we do not show the computations for the values  $\epsilon_0$ ,  $\rho_0$ ,  $|\alpha_{01}^k|$ ,  $|\alpha_{02}^k|$  and  $\frac{w_{max}}{t}$ , which are easily determined for the known quantities of  $\rho_1$ ,  $\rho_2$ ,  $\lambda$ ,  $r$ , and  $e_{ok}$ . Further, we compare the values of the critical pressures  $p_m$  and  $p_k$ , which we derived for different  $\delta$ , with results provided by the linear theory

by constructing the graph of the dependence of the value  $\frac{p R_2^2}{E t^2}$  on  $\delta$ ; see

Figure 2. The formula for the determination of the value of the critical external pressure on the shell according to linear theory for  $\delta \geq 1$  is easily obtained from Reference 2. It has the form

$$p_e = \frac{2E}{\sqrt{3(1-\nu)}} \frac{1}{(\delta-1)} \cdot \frac{t^3}{R_2^2}. \quad [2.7]$$

TABLE 3

$\delta$	1	2	3	4
$\sigma_1$	$1.2225 - 2.6834\epsilon$	$1.2243 - 4.7537\epsilon$	$1.2257 - 5.8618\epsilon$	$1.2267 - 6.5650\epsilon$
$\rho_2$	$1.2225 - 2.6834\epsilon$	$1.2216 - 4.3668\epsilon$	$1.2203 - 5.1142\epsilon$	$1.2208 - 5.5090\epsilon$
$\lambda n$	$4.2590 - 21.165\epsilon$	$4.2581 - 35.559\epsilon$	$4.2558 - 42.089\epsilon$	$4.2562 - 45.572\epsilon$
$\frac{\tau_C^2}{r^2} \cdot \frac{n}{\pi}$	$0.0215 + 4.259\epsilon$ $- 10.583\epsilon^2$	$0.0129 + 4.259\epsilon$ $- 17.685\epsilon^2$	$0.0111 + 4.257\epsilon$ $- 21.048\epsilon^2$	$0.0102 + 4.256\epsilon$ $- 22.788\epsilon^2$
$\epsilon \frac{1}{\pi}$	0.0040	0.0039	0.0042	0.0038
$R_2 \frac{R_2}{e_{ok} t}$	0.2002	0.1942	0.1758	0.1673
$P_k \frac{R_2^2}{Et^2}$	0.4138	0.2840	0.2220	0.2069

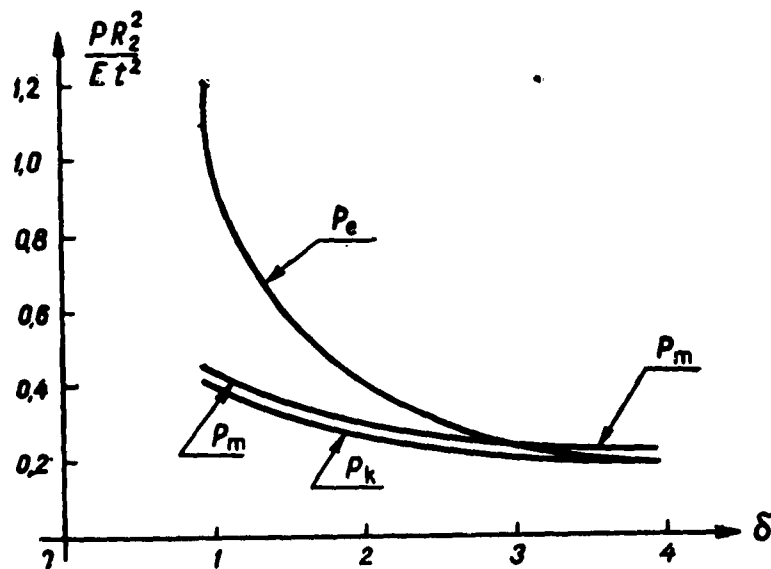


Figure 2

The graph shows that, for the chosen form of displacements, the solution of the problem of local loss of stability of prolate spheroids under external uniform pressure on the shell for  $\delta = \frac{R_1}{R_2} > 3$  cannot be considered as satisfactory since, beginning with  $\delta > 3$ , the magnitude of the critical pressure for which the shell loses its stability, as found from the nonlinear theory,  $p_k$  becomes greater than the value of the upper limit of the critical pressure  $p_e$  as obtained from formula [2.7].

This discrepancy between  $p_k$  and  $p_e$  is explained primarily by the fact that in the choice of the displacement functions we limited ourselves, because of the complexity of the problem, to satisfying only the geometric boundary conditions.

Also, the solution of the problem is influenced by the proper determination of the unknown parameters  $\frac{k}{n}, \frac{k_1}{n}, \frac{k_2}{n}, \frac{k_3}{n^2}, \frac{k_4}{n^2}$ , which in our case

were determined by successive selection only for the case of a sphere and were used for the other particular cases ( $\delta = 2, 3, 4$ ). Finally, we found that the restriction we imposed on the region of buckling, assuming it to be elliptical, apparently had an effect on the solution.

It is necessary to point out that, in the particular case when  $\delta = 1$ , we obtain a fully satisfactory solution to the problem of the local loss of stability of a spherical shell. We refrain from investigating this case, which was satisfactorily discussed at length in Reference 1.

Submitted to the editorial staff  
December 20, 1954

Physico-Technical Institute of  
Kazan Affiliate of the USSR Academy  
of Science

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